On a Discrete-In-Time Order Level Inventory Model for Items with Random Deterioration

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Abstract

The present paper deals with an order level inventory model with continuous in units and discrete-in-time for items having a time dependent random deterioration rate. Shortages are assumed which are fully backlogged. Lastly a numerical example is given to illustrate the model with its sensitivity study.

Keywords: Discrete-in-time, random deterioration, shortages.

Introduction

The effect of deterioration plays an important role in many inventory systems. It is defined as decay, damage or spoilage such that the items can not be used for its original purpose. Food items, photo films, drugs, chemicals etc are few examples of items in which sufficient deterioration may occur during the normal storage period of the units and consequently this loss must be taken into account when analyzing the systems. Effects of time dependent or constant deterioration on inventory models have been investigated by Ghare and Schrader[1], Goel and Aggarwal[2], Covert and Philip[3], Datta and Pal[4], Mandal and Pal[5], Mandal[6] etc. But deterioration of items(drugs, foods) depends upon the fluctuation of humidity, temperature etc and so it is more reasonable and realistic if we assume the deterioration function θ to depend upon a parameter 'α' in addition to time t which ranges over space 'Γ' in which probability density function p(α) is defined. Such type of deterioration function has been developed first by Pal and Mandal[7]. Further development has been made by Mandal[8]

Recently there has been considerable interest in developing mathematical models for describing optimal policies for items in inventory whose utility does not remain the same with the passage of time. Even more time has been considered as a continuous variable which is not exactly the case in practice. In real life problem, time is always treated as a discrete variable, in terms of complete unit of days, weeks, months etc. Dave[9] has developed first time an inventory model for deteriorating items assuming as the time variable is discrete. Further development in this regard has been made by Datta and Pal[10].

In the present paper, an order level inventory model for random deteriorating function has been developed in which time variable is assumed to be discrete one. The deterministic model with instantaneous delivery is considered. This work is generalised by allowing shortages and lastly an example is given to illustrate the model along with its sensitivity analysis.

The Mathematical Model

The mathematical model is developed under the following assumptions:

(i) The demand rate R units per unit time is known and constant
(ii) T is the fixed duration of each production cycle.
(iii) Lead time is zero.
(iv) Replenishment size is constant and its rate is infinite.
(v) Shortages are allowed and fully backlogged.
(vi) The unit cost C, the inventory holding cost C_1 per unit per time unit and the shortage cost C_2 per unit per time unit are known and constant during the period.

(vii) There is no repair or replacement of the deteriorated inventory during a given cycle.
(viii) The fixed lot size q raises the inventory at the beginning of each scheduling period to the order level S.
(ix) A variable fraction $\theta(t,\alpha)$ of the on-hand inventory deteriorates per unit time.

In the present problem, $\theta(t,\alpha)$ is taken as

$\theta(t,\alpha) = \theta_0(\alpha)$ t, (0 < $\theta_0(\alpha)$ <<1)..................(1)

It is some functions of the random variable ' α ' which ranges over a space Γ and in which a probability density function p(α) is defined such that $\int_\Gamma p(\alpha)d\alpha$ =1.

At the time t = 0 of a cycle a batch of q units enters the inventory system, from which (q-S) units are delivered towards backorders leaving S units as the initial inventory level. As time is going on, the inventory level gradually decreases mainly due to demands but partially due to deterioration of units in inventory up to and including time t = t_1 - 1. At t = t_1, the inventory level reaches zero. Further demands for the remaining period (t_1, T) are backlogged.

If I(t) denotes the number of units at the beginning of the time unit t, the difference equations governing the inventory system during the cycle of time T are given by

$$\Delta I(t)+\theta(t,\alpha)I(t)=-R, \quad t = 0,1,2,3................t_1-1 \quad (2)$$

$$\Delta I(t) = - R, \quad t = t_1 ,\ t_1+1 \ ....T \quad (3)$$

Where $\theta(t,\alpha) = \theta_0(\alpha)$ t, (0 < $\theta_0(\alpha)$ <<1)

The solutions of the above difference equations (2) and (3) are found to be the following (neglecting the terms containing square and higher power of $\theta_0(\alpha)$ as 0 < $\theta_0(\alpha)$ <<1)

$$I(t)=(K_1-Rt)[1-\frac{\theta_0(\alpha)t(t-1)}{2}] - \frac{R\theta_0(\alpha)}{6}t(t-1)(t-2) - \frac{R\theta_0(\alpha)}{2}t(t-1), \quad t=0,1,2.......\ t_1-1 \quad (4)$$

And $I(t)=K_2-Rt$, t = t_1 , t_1+1 ....T (5)

Where K_1 and K_2 are constants of integrations.

The boundary conditions are I(0) = S and $I(t_1)$ =0

Therefore using $I(t_1)$ =0, equations (4) and (5) become

$$K_1=\frac{1}{1-\frac{1}{2}\theta_0(\alpha)t_1(t_1-1)}[Rt_1\{1-\frac{\theta_0(\alpha)}{2}t_1(t_1-1)\} + \frac{\theta_0(\alpha)}{6}t_1(t_1-1)(t_2-2) + \frac{\theta_0(\alpha)}{2}t_1(t_1-1) \quad (6)$$

And K_2 = Rt_1 (7)

Again from equation (4), I(0) = S gives $S=K_1$

Therefore from equation (6) and neglecting second and higher powers of $\theta_0(\alpha)$ we get

$$S(\alpha)=R[t_1-\frac{\theta_0(\alpha)}{6}t_1+\frac{\theta_0(\alpha)}{6}t_1^3] \quad (8)$$

Now substituting the values of K_1 and K_2 from the relations (6) and (7) in the equations (4) and (5) respectively, we get the following

$$I(t,\alpha)=R[t_1-\frac{\theta_0(\alpha)}{6}t_1+\frac{\theta_0(\alpha)}{6}t_1^3-\frac{\theta_0(\alpha)}{2}t_1t^2+\frac{\theta_0(\alpha)}{2}t_1t-t$$
$$+\frac{\theta_0(\alpha)}{6}t-\frac{\theta_0(\alpha)}{2}t^2+\frac{\theta_0(\alpha)}{3}t^3],\ t=0,1,2.......\ t_1-1 \qquad (9)$$

And $I(t,\alpha)=R(t_1-t)$, t = t_1 , t_1+1 ....T (10)

(neglecting $O(\theta_0^2(\alpha))$ as 0 < $\theta_0(\alpha)$ <<1)

The average number of units in inventory per unit time during a cycle is

$$H(t_1,\alpha)=\frac{1}{T+1}\sum_{t=0}^{t_1-1}I(t,\alpha)$$

$$=\frac{R}{T+1}[\frac{3-\theta_0(\alpha)}{6}t_1+\frac{6-\theta_0(\alpha)}{12}t_1^2+\frac{\theta_0(\alpha)}{6}t_1^3+\frac{\theta_0(\alpha)}{12}t_1^4]$$ (neglecting $o(\theta_0^2(\alpha))$ as 0 < $\theta_0(\alpha)$ <<1) (11)

The average shortage per unit time during a cycle is

$$G(t_1,\alpha)=-\frac{1}{T+1}\sum_{t=t_1}^{T}I(t,\alpha)$$
$$=\frac{R}{2(T+1)}(T-t_1)(T-t_1+1) \qquad (12)$$

The average number of units that deteriorates per unit time is

$$D(t_1,\alpha)=\frac{1}{T}[S(\alpha)-R\,t_1]$$
$$=\frac{R\theta_0(\alpha)}{6T}(t_1^3-t_1) \qquad (13)$$

Therefore the total average cost of the system per unit time during a cycle is given by

$$K(t_1,\alpha)=C_1H(t_1,\alpha)+C_2G(t_1,\alpha)+C\,D(t_1,\alpha)$$

Hence the mean average total cost of the system per unit time during the cycle is

$$K(t_1)=\langle K(t_1,\alpha)\rangle=C_1\langle H(t_1,\alpha)\rangle+C_2\langle G(t_1,\alpha)\rangle+C\langle D(t_1,\alpha)\rangle \qquad (14)$$

where $\langle K(t_1,\alpha)\rangle=\int_\Gamma K(t_1,\alpha)p(\alpha)d\alpha$

Now $\langle H(t_1,\alpha)\rangle$ = the mean average number of units carrying in inventory per unit time = $\frac{R}{12(T+1)}[2(3-A)t_1+(6-A)t_1^2+2At_1^3+At_1^4]$

$\langle G(t_1,\alpha)\rangle$ = the mean average shortage per unit time = $G(t_1)$

$$=\frac{R}{2(T+1)}(T-t_1)(T-t_1+1)$$

$\langle D(t_1,\alpha)\rangle$ = the mean average amount of inventory that deteriorates per unit time = $\frac{AR}{6T}(t_1^3-t_1)$

where $A=\int_\Gamma\theta_0(\alpha)p(\alpha)d\alpha$ (15) Therefore from (14), substituting the above values we obtain

$$K(t_1) = \frac{C_1R}{12(T+1)}[2(3-A)t_1+(6-A)t_1^2+2At_1^3+At_1^4]+\frac{C_2R}{2(T+1)}(T-t_1)(T-t_1+1) + \frac{CRA}{6T}(t_1^3-t_1) \qquad (16)$$

Since t_1 is a non-negative integer, the conditions for $K(t_1)$ to have an absolute minimum at $t_1=t_1^*$ are

$$\Delta K(t_1^*-1)\le 0\le \Delta K(t_1^*) \qquad (17)$$

and $\Delta^2 K(t_1)\ge 0$, for all t_1 = 0,1,2................T (18)

vide Sasieni et al[11]

Now $\Delta K(t_1)=K(t_1+1)-K(t_1)$

$$= \frac{C_1R}{12(T+1)}[2(3-A)+(6-A)(2t_1+1)+2A(3t_1^2+3t_1+1)+A(4t_1^3+6t_1^2+4t_1+1)] - \frac{C_2R}{(T+1)}(T-t_1) + \frac{CRA}{2T}(t_1^2+t_1) \qquad (19)$$

And $\Delta^2 K(t_1)=\Delta(\Delta K(t_1))$ = $\Delta K(t_1+1)-\Delta K(t_1)$

$$= \frac{RC_1}{12(T+1)}[2(6-A)+36At_1+26A+12At_1^2]+\frac{RC_2}{T+1}+\frac{CAR}{T}(t_1+1)] \qquad (20)$$

Since 0<A<1, we observe that $\Delta^2 K(t_1)>0$ for all t_1 = 0,1,2.............T.

Hence $K(t_1)$ would have an absolute value at $t_1=t_1^*$

if the condition (17) is satisfied.

Now using (17) and (19), the condition for optimality of $K(t_1)$ at $t_1=t_1^*$ becomes

$$M(t_1^*-1)\le C_2T-C_1\le M(t_1^*) \qquad (21)$$

Where

$$M(t_1)=\frac{C_1}{3}(3t_1+2At_1+3At_1^2+At_1^3)+C_2t_1 + \frac{AC}{2T}t_1(T+1)(t_1+1) \qquad (22)$$

Therefore the mean ordering quantity is

$$S(t_1) = \langle S(\alpha)\rangle = R[t_1-\frac{A}{6}t_1+\frac{A}{6}t_1^3] \qquad (23)$$

Special Case:

If the deterioration of the items is switched off the $\theta_0(\alpha)=0$

Then the value of 'A' becomes zero.

In this case the mean average cost equation (16) reduces to the following

$$K(t_1) = \frac{C_1R}{12(T+1)}t_1(t_1+1)+\frac{C_2R}{2(T+1)}(T-t_1)(T-t_1+1) \qquad (24)$$

The value of t_1 for which K(t_1) given by (24)would be minimum satisfying the following inequality

$$\frac{t_1}{T-t_1+1}\le\frac{C_2}{C_1}\le\frac{t_1+1}{T-t_1} \qquad (25)$$

Numerical Illustration

To illustrate the present model we consider the following values of the various parameters :

C_1 = Rs 1.00 per unit per month, C_2 = Rs 9.00 per unit per month

C = Rs 80.00 per unit , R = 200 units per month, T = 12 months

The function $\theta_0(\alpha)$ is taken in the form

$$\theta_0(\alpha) = a + b\alpha \text{ , } 0 < \theta_0(\alpha) << 1, \text{ a, b} > 0$$

where we take a = 0.2, b = 0.1

The probability density function be defined as follows

$$p(\alpha) = \frac{1}{2}(\alpha + 1) \text{ , } -1 \le \alpha \le 1$$

$$= 0, \text{ elsewhere}$$

Using equation (15), we find A=.023

For this system, C_2 T - C_1 = 107

For different discrete values of t_1, the corresponding values of M(t_1) and K(t_1) are given in the Table-1

Table-1

t_1	M(t_1)	K(t_1)
1	30.39	9153.85
2	81.64	7975.28
3^*	154.20^*	7585.13^*
4	248.53	8311.28
5	365.10	10488.72

Analyzing the above table-1 we find the optimum value of t_1 is t_1^* = 3 months and the minimum mean average cost is K(t_1^*) = Rs 7585.13. Using the equation (23) , the optimum value of mean ordering quantity is S(t_1^*) = 784 units.

Sensitivity Analysis and Discussion

We now study the effects of changes in the inventory system parameters such as C_1, C_2, C, a, b and R on the optimal mean average cost K^* = K(t_1^*) and optimal mean ordering quantity S^* = S(t_1^*) in the present EOQ model. The sensitivity analysis is performed by changing each of the parameters by -50%, -20%, +50% and +20% taking one parameter at a time and keeping remaining parameters unchanged.

Table-2: Effects of Changes in the Parameters on the Model

Changing parameters	change(%)	t_1^*	Change of S^*	K^*
C_1	-50	3	784.00	7521.28
	- 20	3	784.00	7559.59
	+20	3	784.00	7610.67
	+50	3	784.00	7648.97
C_2	-50	2	446.00	4167.59
	- 20	3	784.00	6338.97
	+20	3	784.00	8831.28
	+50	3	784.00	10700.51
C	-50	4	1260.00	6777.95
	- 20	3	784.00	7339.79
	+20	3	784.00	7830.46
	+50	2	446.00	8128.62
a	-50	4	1260.00	6931.79
	- 20	3	784.00	7365.64
	+20	3	784.00	7804.62
	+50	2	446.00	8111.69
b	-50	3	784.00	7530.26
	- 20	3	784.00	7623.54
	+20	3	784.00	7640.00
	+50	3	784.00	7804.00
R	-50	3	392.00	3792.56
	- 20	3	627.00	6068.10
	+20	3	940.00	9102.16
	+50	3	1176 .00	11377.69

Comments on the sensitivity analysis

Analyzing the results given in the table-2, the following observations may be made

(i) Increases/decreases with increase/decrease in the value of the system parameter C_1. On the other hand remains unchanged. The results obtained show that S^* is moderately sensitive while K^* is insensitive to the changes in the value of C_1.

(ii) K^* increases/decreases S^* with increase S^* decrease in the value of the K^* system parameter C_2. It can be noticed that changes in the value of is sensitive where as K^* is almost insensitive to the changes in the value of C_2.

(iii). For increase/decrease in the value of the parameter C, the adjoining S^* sensitivity table shows that decreases/increases and K^* increases/ decreases . However it can be seen that S^* and K^* are very sensitive to changes in the value of C.

(iv). The nature of changes of S^* and K^* towards the changes in the value of 'a' are similar as in the changes of the parameter C.

(v). S^* is insensitive and K^* is moderately sensitive to changes in the value of the parameter b.

(vi) K^* and S^* increase/decrease with the increase/decrease in the value of R. The effects on K^* and S^* due to changes in the value of R are very much appreciable.

Concluding Remarks

In this study, we have proposed an inventory model for random deteriorating items with discrete time variable. The method of solving the problem is analytical as well as computational. A numerical example and sensitivity of the solution have been performed in this model.

We have also discussed a special case of the inventory model having no deterioration of items. The proposed discrete nature of time variable is more reasonable and realistic in practice. The sensitivity analysis concludes that the reflection of the unit cost and demand rate on the model are very significant.

Reference

Ghare PM, Schrader G (1963). A model for exponentially decaying inventories, J. Ind. Engg. 14: 283 -243.

Goel VP, Aggarwal SP (1981), Order level inventory system with power demand pattern for deteriorating items, proceeding with All India Seminar on Operational Research and Decision-making, University of Delhi.

Covert R , Philip G (1973). An EOQ model for items with Weibull distribution deterioration. AIIE Trans. 5: 323-326.

Dutta TK , Pal AK (1988). Order Level inventory system with power demand pattern for items with variable rate of deterioration. Ind. J. Pure and Appl . Math. 19 (11):1043-1053.

Mandal B. and Pal A.K.(2000), “Order level inventory system for perishable items with power demand pattern”, Int. J. Mgmt. & Syst., 16(3), 259-276.

Mandal B.(2014). “Optimal inventory management for items with weibull distribution deterioration under alternating demand rates”, Int . J. Eng. and Tech Res 2(4), 359-364.

Pal A.K. and Mandal B.(1998), “ Order level inventory system with power demand pattern for items with random deterioration”, Int J. Mgmt and syst., 14(3), 227-240

Mandal B.(2013), “ An inventory model for random deteriorating tiems with a linear trended in demand and partial backlogging”, Res. J. Bus Mgmt and Accnt., 2(3), 048-052

Dave U.(1979), “on a discrete-in-time order level inventory model for deteriorating items”, JORS, 30, 349-354

Datta T.K. and Pal A.K.(1990), “Order level inventory model for items with time-dependent deterioration rate”, J. Nat. Acad. Maths, 8, 55-60.

Sasieni M., Yaspan A. And Friedman L., “ Operations Research Methods and Problems”, Appendix I¸ John Wiley, new York, 1959.